

The processes $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ at high energies.

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Abstract

According to the helicity conservation (HCns) theorem, the sum of the helicities should be conserved, in any 2-to-2 processes in MSSM with R-parity conservation, at high energies; i.e. all amplitudes violating this rule, must vanish asymptotically. The realization of HCns in $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-, \tilde{\chi}_i^0 \tilde{\chi}_j^0$ is studied, at the one loop electroweak order (EW), and simple high energy expressions are derived for the non-vanishing helicity conserving (HC) amplitudes. These are very similar to the corresponding expressions for $gg \rightarrow W^+W^-, ZZ, \gamma Z, \gamma\gamma$ derived before. Asymptotic relations among observable unpolarized cross sections for many such processes are then obtained, some of which may hold at LHC-type energies.

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1 Introduction

In [1, 2], we have established the helicity conservation (HCns) theorem, to all orders in the minimal supersymmetric model (MSSM) with R-parity conservation. This theorem states that for any 2-to-2 process, the only amplitudes that can survive at asymptotic energies and a fixed angle, are the helicity conserving (HC) ones, in which *the sum of the two initial helicities equals to the sum of the two final ones*. All amplitudes violating this rule, and therefore called helicity violating (HV) amplitudes, are predicted to vanish asymptotically.

In the general all-order proof presented in [1, 2], all mass dimensional parameters were neglected. For a 2-to-2 processes, this is a reasonable assumption for calculating amplitudes at a kinematical region where the SUSY-breaking effects are negligible. Under this assumption, it was then showed that all HV amplitudes vanish asymptotically.

This general proof though, gives no indication on how the asymptotically dominating HC amplitudes behave at high energies [1, 2]. To see this, detail 1loop calculations for specific process are needed.

In such detail calculations, we have observed that in processes involving external gauge bosons, huge cancelations among the various diagrams need to conspire, in order to establish HCns [1, 2, 3]. On the contrary, no such cancelations appear, in processes where all external particles are fermions or scalars. In fact, it was this property that motivated us at first, to look at $ug \rightarrow dW^+$ and $ug \rightarrow \tilde{d}_L \tilde{\chi}_i^+$, at the 1loop EW order [4, 5].

Concerning the asymptotic HC amplitudes for $ug \rightarrow dW^+$ at the 1loop MSSM EW order, we note that they were indeed found to depend on the magnitude of the SUSY masses, and thereby on the SUSY breaking terms [4, 5]. More explicitly, the leading logarithmic corrections were found to depend mainly on the average scale of the SUSY masses; while the subleading energy-independent "constants" were found to depend on ratios of the internal and/or external masses, as well as on the scattering angle [4, 5]. The situation is further complicated when SUSY particles appear in the final state, like e.g. in $ug \rightarrow \tilde{d}_L \tilde{\chi}_i^+$ [5], where the $\tilde{\chi}_i^+$ wave function introduces additional dependencies on ratios of the SUSY breaking mass-dimensional parameters.

The 1loop EW order calculations for $ug \rightarrow dW$ and¹ $ug \rightarrow \tilde{d}_L \tilde{\chi}_i^+$, were also used to derive simple asymptotic relations between the unpolarized cross sections for these processes [4, 5]. The particularly interesting thing in these relations is that they may be satisfied even at LHC type energies, if the SUSY scale is within the rather wide range of the benchmarks [6, 7, 8, 9, 10].

Subsequently we studied, at the same 1loop EW order, the processes $gg \rightarrow HH'$, VH , VV' , where two gluons fuse to produce Higgs-type scalars or electroweak vector particles ($V, V' = W, Z, \gamma$), in either SM or MSSM [11, 12]. In this case, there are no Born contributions, neither any gauge contribution; and no large logarithmic contributions to the HC amplitudes appear asymptotically. As a result, the asymptotic HC amplitudes were found to be very sensitive to the differences between the SM and MSSM dynamics; but quite insensitive to the SUSY breaking mass terms. Thus, examples were found,

¹Here \tilde{d}_L describe a down-L-squark and $\tilde{\chi}_i^+$ a chargino.

involving either longitudinal [11], or transverse gauge bosons [12], where HCns is strongly violated in SM, while obeyed in MSSM. This shows that HCns is indeed a genuine SUSY property, drastically reducing the number of the asymptotically non-vanishing amplitudes [1, 2, 13].

The purpose of the present work is to study the helicity amplitudes for $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$, at the 1loop EW order in MSSM. In these processes, two gluons fuse to produce two charginos or neutralinos, which constitute the supersymmetric transformed of appropriate combinations of the final states in $gg \rightarrow HH', VH, VV'$ studied in [11, 12]. This leads to interesting relations among the corresponding processes at high energies.

Again, very simple expressions for the asymptotic HC amplitudes for $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ are obtained, similar to the corresponding $gg \rightarrow VV'$ amplitudes found in [12]. The interrelations between the two processes, naturally appear in two different forms; the gauge-gaugino relations, and the Goldstone-higgsino ones. In all cases, they are independent of the squark masses running along the internal lines of the contributing diagrams. In addition, fascinating SUSY identities between the asymptotic amplitudes for $gg \rightarrow VV'$ and $\tilde{g}\tilde{g} \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ are noted.

Furthermore, simple asymptotic relations among the unpolarized cross sections for $gg \rightarrow VV'$ and $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ are constructed, analogous to those in [5]. It is found that some these relations may be approximately respected, even at LHC-type energies.

A Fortran code supplying the 1loop helicity amplitudes for $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$, at any energy and angle in MSSM, is herewith released [14].

The contents of the paper are the following: Sect.2 is devoted to the amplitude analysis of $gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j$, including a presentation of the numerical code mentioned above. In Sect.3, the high energy expressions for these amplitudes are presented, which, together with those for $gg \rightarrow VV'$, are used to illustrate the helicity conservation theorem for these processes, and to derive the aforementioned asymptotic relations among their unpolarized cross sections. The numerical results are presented in Sect. 4. The concluding remarks appear in Section 5.

2 The $gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$, $\tilde{\chi}_i^+ \tilde{\chi}_j^-$ amplitudes

Defining the usual kinematical variables

$$s = (p_g + p'_g)^2 \quad , \quad t = (p_g - p_{\tilde{\chi}_i})^2 \quad , \quad u = (p_g - p_{\tilde{\chi}_j})^2 \quad , \quad (1)$$

in terms of the incoming and outgoing momenta, the helicity amplitudes are denoted as

$$F(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)_{\mu\mu'\tau\tau'} \equiv F_{\mu\mu'\tau\tau'}^{ij}(\sqrt{s}, \theta) \quad , \quad (2)$$

where s is the squared c.m. energy, and θ is the c.m. scattering angle between p_g and $p_{\tilde{\chi}_i^0}$. The indices μ, μ', τ, τ' in (2) denote respectively the $g, g, \tilde{\chi}_i, \tilde{\chi}_j$ helicities, using the

standard conventions [15]; while i, j describe the mass eigenstates of the final² neutralinos or charginos. A color-factor δ^{ab} , where a, b are the gluon color indices, is always removed from the amplitude, defined so that the iF -phase coincides with the phase of the S-matrix. The c.m. momentum of the final state particles is

$$p = \frac{[s - (m_i + m_j)^2]^{\frac{1}{2}}[s - (m_i - m_j)^2]^{\frac{1}{2}}}{2\sqrt{s}} . \quad (3)$$

The unpolarized differential cross section, as well the dimensionless $\tilde{\sigma}$, are defined as

$$\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j) \equiv \frac{512\pi}{\alpha^2 \alpha_s^2} \frac{s^{3/2}}{p} \frac{d\sigma(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j; s)}{d \cos \theta} = \frac{\sum_{\mu\mu'\tau\tau'} |F_{\mu\mu'\tau\tau'}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)|^2}{\alpha^2 \alpha_s^2} , \quad (4)$$

where the summation is over all initial and final helicities. For identical neutralinos $\tilde{\chi}_i^0 \tilde{\chi}_i^0$, the integrated cross sections in the region $-1 < \cos \theta < 1$, must be divided by 2.

Bose and Fermi statistics for the initial gluons and the final neutralinos constrain the helicity amplitudes by

$$gg \Rightarrow F_{\mu\mu'\tau\tau'}^{ij}(\theta) = (-1)^{\tau-\tau'} F_{\mu'\mu\tau\tau'}^{ij}(\pi - \theta) , \quad (5)$$

$$\tilde{\chi}_i^0 \tilde{\chi}_j^0 \Rightarrow F_{\mu\mu'\tau\tau'}^{ij}(\theta) = F_{\mu\mu'\tau'\tau}^{ji}(\pi - \theta) . \quad (6)$$

In addition, when neglecting the CP violating contribution to MSSM, we also obtain

$$F_{-\mu, -\mu', -\tau, -\tau'}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0}(\theta) = (-1)^{(\tau-\tau')} \eta_i \eta_j F_{\mu\mu'\tau\tau'}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0}(\theta) = (-1)^{(\tau-\tau')} F_{-\mu', -\mu, -\tau', -\tau}^{\tilde{\chi}_j^0 \tilde{\chi}_i^0}(\theta) , \quad (7)$$

$$F_{\mu\mu'\tau\tau'}^{\tilde{\chi}_i^+ \tilde{\chi}_j^-}(\theta) = F_{-\mu', -\mu, -\tau', -\tau}^{\tilde{\chi}_j^+ \tilde{\chi}_i^-}(\theta) . \quad (8)$$

According to the HCNs theorem, the helicity conserving HC amplitudes, which satisfy

$$\mu + \mu' = \tau + \tau' , \quad (9)$$

will be the only ones surviving asymptotically. These are

$$F_{+-+-}^{ij}(\theta) , \quad F_{+--+}^{ij}(\theta) , \quad F_{-++-}^{ij}(\theta) , \quad F_{-+-+}^{ij}(\theta) , \quad (10)$$

constrained by (5) as

$$F_{+--+}^{ij}(\theta) = -F_{-++-}^{ij}(\pi - \theta) , \quad F_{-+-+}^{ij}(\theta) = -F_{+-+-}^{ij}(\pi - \theta) . \quad (11)$$

All helicity violating (HV) amplitudes, which by definition violate (9), must vanish in the high energy limit.

²In calculating these amplitudes, positive (negative) energy Dirac wave functions are always used for describing the first (second) outgoing particles $\tilde{\chi}_i$ ($\tilde{\chi}_j$).

On the basis of (5-8), the independent HC and HV amplitudes for $gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$, may be chosen as

$$\begin{aligned} HC &\Rightarrow F_{-+-+}^{ij}(\theta) , F_{-++-}^{ij}(\theta) , \\ HV &\Rightarrow F_{----}^{ij}(\theta) , F_{---+}^{ij}(\theta) , F_{--+ -}^{ij}(\theta) , F_{--+ +}^{ij}(\theta) , \\ &F_{-+--}^{ij}(\theta) , F_{-+++}^{ij}(\theta) . \end{aligned} \quad (12)$$

Correspondingly for $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, the independent amplitudes are chosen as

$$\begin{aligned} HC &\Rightarrow F_{-+-+}^{ij}(\theta) , F_{-++-}^{ij}(\theta) , F_{+--+}^{ij}(\theta) , F_{+-- -}^{ij}(\theta) , \\ HV &\Rightarrow F_{----}^{ij}(\theta) , F_{---+}^{ij}(\theta) , F_{--+ -}^{ij}(\theta) , F_{--+ +}^{ij}(\theta) , F_{-+--}^{ij}(\theta) , F_{-+++}^{ij}(\theta) , \\ &F_{+ + - -}^{ij}(\theta) , F_{+ + - +}^{ij}(\theta) , F_{+ + + -}^{ij}(\theta) , F_{+ + + +}^{ij}(\theta) , F_{+ - - -}^{ij}(\theta) , F_{+ - - +}^{ij}(\theta) . \end{aligned} \quad (13)$$

The contributing diagrams to the 1loop EW order, appearing in Fig.1, are calculated analytically, in terms of Passarino-Veltman (PV) functions [16]. To the contribution of the triangular graphs A, A', B, B' and the boxes G, F, H , we also add the gluon symmetrization (gSYM) contribution, obtained through the interchange

$$\mu \leftrightarrow \mu' , \quad \cos \theta \leftrightarrow -\cos \theta , \quad \sin \theta \leftrightarrow -\sin \theta . \quad (14)$$

The contributions of the graphs C, C', D are already gluon-symmetrized.

Using these, the Fortran code ggXXcode has been constructed giving $F_{\mu\mu'\tau\tau'}^{ij}(\sqrt{s}, \theta)$, in terms of the c.m. energy in TeV and the c.m. angle in radians [14]. A factor $\alpha\alpha_s$ (together with δ^{ab}) is always removed from the amplitudes given by the code. All input parameters are assumed at the electroweak scale, and all mass-dimensional parameters are in TeV. The quark masses of the first two generations are neglected, as well as CP violation, so that all input parameters are real. The PV functions are calculated using the looptools subroutines [17, 18]. The results of the codes are contained in output files specified as ".dat". An accompanying Readme, fully explains the compilation of the code [14].

3 High energy behavior in MSSM

Before addressing the $gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0, \tilde{\chi}_i^+ \tilde{\chi}_j^-$ asymptotic amplitudes; we recapitulate the corresponding expressions for the $gg \rightarrow VV'$ amplitudes $F_{\mu\mu'\tau\tau'}^{VV'}$, with $VV' = ZZ, \gamma Z, \gamma\gamma, W^+W^-$, and τ, τ' this time describing the V, V' helicities [12].

Abiding with the HCNs theorem, only the HC amplitudes can be non vanishing asymptotically, which implies that V, V' can be either both transverse, or both longitudinal. For

presenting them, it is convenient to define

$$\tilde{\delta}\left(\frac{x}{y}\right) = -4 \left[\ln^2 \left(\frac{x}{y} \right) + \pi^2 \right] , \quad (15)$$

with x and y being complex, and the standard branches for the logarithms in the complex plane are used. Using this, we define [12]

$$\begin{aligned} \delta_{+-+} &= \delta_{-++} \equiv \delta^t = \tilde{\delta}\left(\frac{t+i\epsilon}{s+i\epsilon}\right) \\ &= -4 \left[\ln^2 \left(\frac{2}{1-\cos\theta} \right) - i2\pi \ln \left(\frac{2}{1-\cos\theta} \right) \right] , \end{aligned} \quad (16)$$

$$\begin{aligned} \delta_{+--} &= \delta_{-+-} \equiv \delta^u = \tilde{\delta}\left(\frac{u+i\epsilon}{s+i\epsilon}\right) \\ &= -4 \left[\ln^2 \left(\frac{2}{1+\cos\theta} \right) - i2\pi \ln \left(\frac{2}{1+\cos\theta} \right) \right] , \end{aligned} \quad (17)$$

$$\delta_{++++} = \delta_{----} \equiv \delta^{tu} = \tilde{\delta}\left(\frac{t+i\epsilon}{u+i\epsilon}\right) = -4 \left[\ln^2 \left(\frac{1+\cos\theta}{1-\cos\theta} \right) + \pi^2 \right] , \quad (18)$$

which suffice to describe all asymptotic HC amplitudes considered here. Note that for asymptotic energies, the relations $t = -s(1 - \cos\theta)/2$ and $u = -s(1 + \cos\theta)/2$ are consistently used.

Using these, the transverse gauge asymptotic HC amplitudes ($\tau\tau' \neq 0$), become [12, 19, 20]

$$\begin{aligned} F(gg \rightarrow ZZ)_{\mu\mu'\tau\tau'}^{\text{as}} &= \alpha\alpha_s \frac{(9 - 18s_W^2 + 20s_W^4)}{24s_W^2 c_W^2} \delta_{\mu\mu'\tau\tau'} , \\ F(gg \rightarrow \gamma Z)_{\mu\mu'\tau\tau'}^{\text{as}} &= \alpha\alpha_s \frac{(9 - 20s_W^2)}{24s_W c_W} \delta_{\mu\mu'\tau\tau'} , \\ F(gg \rightarrow \gamma\gamma)_{\mu\mu'\tau\tau'}^{\text{as}} &= \alpha\alpha_s \frac{5}{6} \delta_{\mu\mu'\tau\tau'} , \\ F(gg \rightarrow W^+W^-)_{\mu\mu'\tau\tau'}^{\text{as}} &= \alpha\alpha_s \frac{3}{8s_W^2} \delta_{\mu\mu'\tau\tau'} , \end{aligned} \quad (19)$$

while the longitudinal ones ($\tau = \tau' = 0$) are³

$$\begin{aligned} F(gg \rightarrow ZZ)_{+-00}^{\text{as}} &= F(gg \rightarrow ZZ)_{-+00}^{\text{as}} = \alpha\alpha_s \frac{(m_t^2 + m_b^2)}{16s_W^2 m_W^2} \left\{ \frac{\delta^t(1 - \cos\theta)}{1 + \cos\theta} + \frac{\delta^u(1 + \cos\theta)}{1 - \cos\theta} \right\} , \\ F(gg \rightarrow W^+W^-)_{+-00}^{\text{as}} &= \frac{\alpha\alpha_s}{8s_W^2 m_W^2} \left\{ \frac{m_b^2 \delta^t(1 - \cos\theta)}{1 + \cos\theta} + \frac{m_t^2 \delta^u(1 + \cos\theta)}{1 - \cos\theta} \right\} , \\ F(gg \rightarrow W^+W^-)_{-+00}^{\text{as}} &= \frac{\alpha\alpha_s}{8s_W^2 m_W^2} \left\{ \frac{m_b^2 \delta^u(1 + \cos\theta)}{1 - \cos\theta} + \frac{m_t^2 \delta^t(1 - \cos\theta)}{1 + \cos\theta} \right\} . \end{aligned} \quad (20)$$

³The quark masses of the first two generations are neglected.

For later use, we also deduce from (19) the asymptotic amplitudes for the transverse W_μ and B_μ gauge production

$$\begin{aligned} F(gg \rightarrow W^+W^-)_{\mu\mu'\tau\tau'}^{\text{as}} &= F(gg \rightarrow W^{(3)}W^{(3)})_{\mu\mu'\tau\tau'}^{\text{as}} = \frac{3\alpha\alpha_s}{8s_W^2} \delta_{\mu\mu'\tau\tau'} \quad , \\ F(gg \rightarrow BB)_{\mu\mu'\tau\tau'}^{\text{as}} &= \frac{11\alpha\alpha_s}{24c_W^2} \delta_{\mu\mu'\tau\tau'} \quad , \end{aligned} \quad (21)$$

where (15-18) are used.

As seen from (19, 20), the HC asymptotic amplitudes for $gg \rightarrow VV'$ are independent of the SUSY parameters. For neutral gauge bosons, they have been first obtained in [19], and the proof was extended to the W^+W^- case in [12]. In both cases, the proof was through very lengthy computations.

For transverse gauge bosons, a much simpler way to verify the result (21), is by studying the supersymmetry-transformed processes describing the high energy gluino fusion to gauginos; i.e. $\tilde{g}\tilde{g} \rightarrow \tilde{V}\tilde{V}'$ with $VV' = W^+W^-$, $W^{(3)}W^{(3)}$, BB . At the 1loop EW order, the asymptotically non vanishing HC amplitudes, are then found to satisfy [21]

$$(-1)^{\tilde{\mu}-\tilde{\tau}'} F(\tilde{g}\tilde{g} \rightarrow \tilde{V}\tilde{V}')_{\tilde{\mu}\tilde{\mu}'\tilde{\tau}\tilde{\tau}'} = F(gg \rightarrow VV')_{\mu\mu'\tau\tau'} \quad , \quad (22)$$

where $\tilde{\mu}$, $\tilde{\mu}'$, $\tilde{\tau}$, $\tilde{\tau}'$ are the gluino and gaugino helicities, which of course receive half integers values. As seen in (22), most of the gauge and gaugino asymptotic amplitudes, are identical. But for $\tilde{\mu} - \tilde{\tau}' = \pm 1$, sign differences appear which must be related to the way we define the gluino amplitudes, where positive energy Dirac wave functions are used to describe the first incoming or outgoing fermionic particle, and negative energy Dirac wave functions are used for describing the second one.

The validity of (22) for transverse VV' , is of course a consequence of the fact that, at the 1loop EW level, the high energy HC amplitudes for $gg \rightarrow VV'$ are independent of the SUSY breaking parameters, while in the corresponding $\tilde{g}\tilde{g} \rightarrow \tilde{V}\tilde{V}'$, the gaugino mixing is ignored. An analogous result for $ug \rightarrow dW$ and its SUSY transformed process, could only be approximately true; since some dependence of the HC amplitudes on the SUSY breaking masses remains in this case, even at asymptotic energies [5, 4].

In analogy to (22), the corresponding asymptotic relations between the longitudinal vector boson amplitudes and the gluino-to-higgsino ones, may be given by first noting that

$$\begin{aligned} F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{\pm\mp\pm\mp} &= \frac{\alpha\alpha_s}{8} \frac{m_b^2}{m_W^2 s_W^2 \cos^2 \beta} \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \delta^t \quad , \\ F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{\mp\pm\pm\mp} &= -\frac{\alpha\alpha_s}{8} \frac{m_b^2}{m_W^2 s_W^2 \cos^2 \beta} \frac{(1 + \cos \theta)}{(1 - \cos \theta)} \delta^u \quad , \\ F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{\pm\mp\pm\mp} &= \frac{\alpha\alpha_s}{8} \frac{m_t^2}{m_W^2 s_W^2 \sin^2 \beta} \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \delta^t \quad , \\ F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{\mp\pm\pm\mp} &= -\frac{\alpha\alpha_s}{8} \frac{m_t^2}{m_W^2 s_W^2 \sin^2 \beta} \frac{(1 + \cos \theta)}{(1 - \cos \theta)} \delta^u \quad , \end{aligned}$$

$$\begin{aligned}
F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^+ \tilde{H}_1^-)_{+-+-} &= \frac{\alpha\alpha_s}{8} \frac{m_b^2}{m_W^2 s_W^2 \cos^2 \beta} \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \delta^t, \\
F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^+ \tilde{H}_2^-)_{+-+-} &= \frac{\alpha\alpha_s}{8} \frac{m_t^2}{m_W^2 s_W^2 \sin^2 \beta} \frac{(1 - \cos \theta)}{(1 + \cos \theta)} \delta^t,
\end{aligned} \tag{23}$$

at 1loop EW order, where (16,17) are used. Comparing these to (20) we obtain the asymptotic relations

$$\begin{aligned}
F(gg \rightarrow ZZ)_{+-00}^{\text{as}} &= \frac{\sin^2 \beta}{2} \left[F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{+-+-} - F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{----} \right] \\
&\quad + \frac{\cos^2 \beta}{2} \left[F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{+-+-} - F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{----} \right], \\
F(gg \rightarrow ZZ)_{-+00}^{\text{as}} &= \frac{\sin^2 \beta}{2} \left[F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{-++-} - F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^0 \tilde{H}_2^0)_{-+-+} \right] \\
&\quad + \frac{\cos^2 \beta}{2} \left[F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{-++-} - F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^0 \tilde{H}_1^0)_{-+-+} \right], \\
F(gg \rightarrow W^+W^-)_{+-00}^{\text{as}} &= \cos^2 \beta F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^+ \tilde{H}_1^-)_{+-+-} - \sin^2 \beta F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^+ \tilde{H}_2^-)_{----}, \\
F(gg \rightarrow W^+W^-)_{-+00}^{\text{as}} &= -\cos^2 \beta F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_1^+ \tilde{H}_1^-)_{-++-} \\
&\quad + \sin^2 \beta F(\tilde{g}\tilde{g} \rightarrow \tilde{H}_2^+ \tilde{H}_2^-)_{-+-+},
\end{aligned} \tag{24}$$

in agreement with the SUSY transformations relating the higgsinos to the G^\pm , G^0 Goldstone bosons, and the longitudinal EW vector bosons, through the equivalence theorem [22]. Note that in contrast to (22, 21) which are independent of any symmetry breaking parameter, some symmetry breaking appears in (24), contained in the angle β .

The actual form of (22) and (24) is an impressive example of how contrived supersymmetry can be. Without SUSY, we would naively expect that the angular dependencies of amplitudes involving particle of different spin, cannot be the same; particularly for the F_{+-+-} amplitudes in (22), where the⁴ d^J functions involved in the partial wave expansion are very different [15]. It is amusing to see how SUSY manages to keep the amplitudes unchanged, after all partial waves are summed over. A genuine SUSY property indeed, not shared by any other symmetry in particle physics!

We next turn to the asymptotic amplitudes for $gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0, \tilde{\chi}_i^+ \tilde{\chi}_j^-$; with τ, τ' now describing the neutralino and chargino helicities. These final states are the supersymmetric transformed of the final states in (19, 20). As expected, only the HC amplitudes survive asymptotically, while the HV ones vanish at very high energies.

The contributions of the diagrams of Fig.1, to the various HV amplitudes at high energies [21], are as follows:

- The HV amplitudes for $\mu = \mu'$ and $\tau = \tau'$, vanish asymptotically like $\sim m/\sqrt{s}$, without any logarithmic corrections.

⁴Remember that all helicities in the l.h.s. of (22) are $\pm 1/2$, while in the r.h.s are ± 1 .

- The HV amplitudes for $\mu = \mu'$ and $\tau = -\tau'$ are strongly suppressed like $\sim m^2/s$, again with no logarithmic corrections. These amplitudes receive their main contributions from the boxes F, G, H .
- Finally, the HV amplitudes for $\mu = -\mu'$ and $\tau = \tau'$, receive high energy contributions of the form

$$-\frac{\alpha\alpha_s}{4}\left(\frac{2m_q}{\sqrt{s}}\right)\left[\ln^2\left(\frac{t}{m_q^2}\right)+\ln^2\left(\frac{u}{m_q^2}\right)-\ln^2\left(\frac{s}{m_q^2}\right)\right], \quad (25)$$

arising from third generation quark-squark boxes, where m_q is the top or bottom mass. These are the slowest vanishing HV amplitudes, at high energies.

We next turn to the HC amplitudes for $gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j$, which necessarily satisfy $\mu = -\mu'$ and $\tau = -\tau'$, and behave asymptotically like angular dependent "constants", mainly feeded by the F, G and H boxes of Fig.1. In all cases, the approach to the asymptotic values is determined by power laws, possibly corrected by logarithms. For charginos these are

$$\begin{aligned} F(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)_{-++-}^{as} &= \frac{\alpha\alpha_s \delta^t (1 - \cos \theta)}{8s_W^2 \sin \theta} \left\{ 3Z_{1i}^{+*} Z_{1j}^+ + Z_{2i}^{+*} Z_{2j}^+ \frac{m_t^2}{m_W^2 \sin^2 \beta} \right\}, \\ F(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)_{+-+-}^{as} &= -\frac{\alpha\alpha_s \delta^t (1 - \cos \theta)}{8s_W^2 \sin \theta} \left\{ 3Z_{1i}^- Z_{1j}^{*-} + Z_{2i}^- Z_{2j}^{*-} \frac{m_b^2}{m_W^2 \cos^2 \beta} \right\}, \\ F(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)_{+--+}^{as} &= -\frac{\alpha\alpha_s \delta^u (1 + \cos \theta)}{8s_W^2 \sin \theta} \left\{ 3Z_{1i}^{+*} Z_{1j}^+ + Z_{2i}^{+*} Z_{2j}^+ \frac{m_t^2}{m_W^2 \sin^2 \beta} \right\}, \\ F(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)_{-++-}^{as} &= \frac{\alpha\alpha_s \delta^u (1 + \cos \theta)}{8s_W^2 \sin \theta} \left\{ 3Z_{1i}^- Z_{1j}^{*-} + Z_{2i}^- Z_{2j}^{*-} \frac{m_b^2}{m_W^2 \cos^2 \beta} \right\}, \end{aligned} \quad (26)$$

while for neutralinos

$$\begin{aligned} F(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{-++-}^{as} &= F(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{-++-}^{as*} = -F(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{+-+-}^{as} = \alpha\alpha_s \frac{\delta^t (1 - \cos \theta)}{\sin \theta} \\ &\cdot \left\{ Z_{1i}^N Z_{1j}^{N*} \frac{11}{24c_W^2} + Z_{2i}^N Z_{2j}^{N*} \frac{3}{8s_W^2} + Z_{3i}^N Z_{3j}^{N*} \frac{m_b^2}{8s_W^2 m_W^2 \cos^2 \beta} + Z_{4i}^N Z_{4j}^{N*} \frac{m_t^2}{8s_W^2 m_W^2 \sin^2 \beta} \right\}, \\ F(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{-++-}^{as} &= -F(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)_{+-+-}^{as*} = -F(gg \rightarrow \tilde{\chi}_j^0 \tilde{\chi}_i^0)_{+-+-}^{as} = \alpha\alpha_s \frac{\delta^u (1 + \cos \theta)}{\sin \theta} \\ &\cdot \left\{ Z_{1i}^N Z_{1j}^{N*} \frac{11}{24c_W^2} + Z_{2i}^N Z_{2j}^{N*} \frac{3}{8s_W^2} + Z_{3i}^N Z_{3j}^{N*} \frac{m_b^2}{8s_W^2 m_W^2 \cos^2 \beta} + Z_{4i}^N Z_{4j}^{N*} \frac{m_t^2}{8s_W^2 m_W^2 \sin^2 \beta} \right\}, \end{aligned} \quad (27)$$

where $Z_{\alpha i}^-$, $Z_{\alpha i}^+$ and $Z_{\alpha i}^N$ are the chargino and neutralino mixing matrices respectively [22].

Technically, the asymptotic expressions (26,27) were first obtained by studying gluon-fusion to pairs of bino, wino and higgsino components. Afterwards, the final expressions for any physical $gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j$ process were constructed, by introducing the corresponding Z mixing-matrix elements.

The asymptotic expressions for the dimensionless differential cross sections $\tilde{\sigma}$ defined in (4), for chargino-neutralino and vector-boson production are given by

$$\begin{aligned}\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)^{as} &\equiv \frac{\sum_{HC} |F(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)_{\mu\mu'\tau\tau'}^{as}|^2}{\alpha^2 \alpha_s^2}, \\ \tilde{\sigma}(gg \rightarrow VV')^{as} &\equiv \frac{\sum_{HC} |F(gg \rightarrow VV')_{\mu\mu'\tau\tau'}^{as}|^2}{\alpha^2 \alpha_s^2},\end{aligned}\quad (28)$$

where the summation is over the asymptotic HC amplitudes in (19, 20, 26, 27).

As seen from (26,27), the sizes of the gaugino and higgsino parts of the HC amplitudes are solely determined by the Z mixing-matrix elements, multiplied by coefficients which, for large $\tan\beta$, acquire similar magnitudes; compare the terms within the curly brackets in (26,27). So the SUSY benchmark dependence of the asymptotic HC amplitudes and $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)^{as}$ will be controlled by the sizes of the Z -matrix elements and the β -angle.

In the "inclusive" asymptotic cross section $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)^{as}$ though, where all chargino or neutralino final states are summed over, the unitarity of the Z -matrices eliminates the Z -dependence, so that the angle β is the sole SUSY parameter on which this quantity depends.

Combining (4, 28), this implies that the energy- and angle-dependent quantities

$$\frac{\tilde{\sigma}(gg \rightarrow W^+W^-)}{\tilde{\sigma}(gg \rightarrow W^+W^-)^{as}}, \frac{\tilde{\sigma}(gg \rightarrow ZZ)}{\tilde{\sigma}(gg \rightarrow ZZ)^{as}}, \frac{\tilde{\sigma}(gg \rightarrow \gamma Z)}{\tilde{\sigma}(gg \rightarrow \gamma Z)^{as}}, \frac{\tilde{\sigma}(gg \rightarrow \gamma\gamma)}{\tilde{\sigma}(gg \rightarrow \gamma\gamma)^{as}}, \quad (29)$$

$$\frac{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)}{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}}, \frac{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)}{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}}, \quad (30)$$

$$\frac{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)}{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}}, \frac{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)}{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}}, \quad (31)$$

should all approach 1, as the energy reaches sufficient process dependent values.

Note that the denominators $\tilde{\sigma}^{as}$ in (29) for gauge production, only depend on the gauge couplings and are independent of the SUSY benchmark; compare (19, 20). As already said, the only SUSY dependence of the denominators in (30), is contained in the angle β ; while the SUSY dependence of the denominators of the two quantities in (31), also involves Z -matrix elements.

In the next Section we will see how the various quantities in (29, 30, 31) approach unity, as the energy increases.

4 Numerical results

In this section we give numerical illustrations for the $gg \rightarrow \tilde{\chi}_1 \tilde{\chi}_2$ amplitudes. As already said, all HV amplitudes vanish asymptotically, while the asymptotic HC amplitudes are

Table 1: Asymptotic HC amplitudes for $gg \rightarrow \tilde{\chi}_1 \tilde{\chi}_2$ divided by $\alpha\alpha_s$, and $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1 \tilde{\chi}_2)^{as}$ at $\theta = 60^\circ$, in $SPS1a'$ [6].

$gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$		$gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$	
$F_{-++-}(\theta)$	$0.21 - i0.94$	$F_{-++-}(\theta)$	$0.8 - i3.6$
$F_{+-+-}(\theta)$	$-0.21 + i0.94$	$F_{+-+-}(\theta)$	$2.1 - i9.6$
$F_{+--+}(\theta)$	$-0.027 + i0.59$	$F_{+--+}(\theta)$	$-0.10 + i2.3$
$F_{-+-+}(\theta)$	$0.027 - i0.59$	$F_{-+-+}(\theta)$	$-0.27 + i6.0$

given by (26, 27). Using these, we give in Table 1, the HC asymptotic amplitudes at $\theta = 60^\circ$ for the benchmark $SPS1a'$ [6]. Needless to say, that these results fully agree with those obtained from the complete 1loop code at high energy [14].

We next turn to asymptotic dimensionless differential cross sections $\tilde{\sigma}^{as}$ defined in (28) and compare the asymptotic production of the specific neutralino or chargino pair with ($i = 1, j = 2$), with the case where summation over all charginos or neutralinos is done. As said above, in the first case strong SUSY benchmark dependence appears, while in the second case, the only SUSY dependence is through the angle β . As SUSY benchmarks we select five constrained MSSM models defined in Table 2, where the grand scale $\tan\beta$ varies from 10 to 50; while the bino, wino and higgsino components of the lightest neutralinos and chargino cover a wide range of possibilities.

Table 2: Input parameters at the grand scale, for five constrained MSSM benchmark models with $\mu > 0$. All dimensional parameters in GeV.

	$SPS1a'$ [6]	mSP4 [9]	BBSSW [8]	AD1 [24]	BKPU [23]
$m_{1/2}$	250	137	900	900	2900
m_0	70	1674	4716	400	8700
A_0	-300	1985	0	0	0
$\tan\beta$	10	18.6	30	40	50

In addition, we consider the non-universal AD2 model suggested in [24], again characterized by $\mu > 0$, but having unequal Higgs masses at the grand scale. Its defining high scale parameters are

$$\begin{aligned}
 \text{AD2 - model} \quad M_1 = M_2 = M_3 = A_0 = 420, \quad \tan\beta = 40, \\
 m_0 = 500, \quad m_{H_u}^2 = 6 \cdot 10^5, \quad m_{H_d}^2 = 3.6 \cdot 10^5,
 \end{aligned} \tag{32}$$

where all dimensional parameters are in GeV. Note that the grand scale values for $\tan\beta$ in AD1 and AD2, are the same.

The lightest neutralino $\tilde{\chi}_1^0$ in these models is mostly a bino for $SPS1a'$, mSP4, BBSSW, AD1; and a higgsino for AD2, BKPU. Correspondingly $\tilde{\chi}_2^0$ is mostly a wino for $SPS1a'$,

mSP4, AD1; and a higgsino for BBSSW, AD2, BKPU. Finally $\tilde{\chi}_1^+$ is mainly a wino for *SPS1a'*, mSP4, AD1; and a higgsino for BBSSW, AD2, BKPU.

The corresponding results for $\tilde{\sigma}^{as}$ are given in Table 3, where of course the EW scale of the various SUSY parameters are used.

Table 3: Asymptotic dimensionless cross sections for $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1 \tilde{\chi}_2)^{as}$ and $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)^{as}$ at $\theta = 60^\circ$, for the benchmarks of Table 2 and AD2.

	$\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)^{as}$	$\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}$	$\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)^{as}$	$\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}$
<i>SPS1a'</i>	2.2	11294	156	6961
mSP4	1.2	11506	31	7066
BBSSW	66	13488	62	8058
AD1	0.7	19809	38	11218
AD2	35	19791	77	11209
BKPU	776	31617	4.6	17122

As seen from Table 3, the results for $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1 \tilde{\chi}_2)^{as}$ vary from model to model, in a more or less random way, depending on the many SUSY parameters affecting the relevant Z-matrix elements. In contrast to this, the benchmark dependence largely disappears in $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j)^{as}$, with only the dependence on the EW β -value remaining⁵; compare (26, 27) and (28). In all cases, irrespective of the nature of the lightest neutralinos or charginos, the dimensionless cross sections satisfy

$$\begin{aligned}
\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0) &\gg \tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0) , \\
\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-) &\gg \tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-) ,
\end{aligned} \tag{33}$$

at asymptotic energies [6, 7, 8, 9, 10, 23, 24]. It may be worthwhile to remark though, that for the higgsino $\tilde{\chi}_1^0, \tilde{\chi}_2^0$ cases of the models AD2 and BKPU, these inequalities are somewhat less strong. The physical reason for these strong inequalities should be due to the orthogonality of the $(\tilde{\chi}_1, \tilde{\chi}_2)$ states and the $SU(2)$ EW gauge symmetry of MSSM. This may be realized by contemplating on the structure of (26,27), which also leads to the conclusion that at least some of the diagonal production cross sections $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_i \tilde{\chi}_i)$, should be comparable to the summed quantities in the l.h.s. of (33). Exactly which, is of course model-dependent. For *SPS1a'* we have checked these statements numerically.

We next turn to the exact 1loop EW order amplitudes, for $gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ and $gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$, calculated from the ggXXcode [14] and given respectively in Figs.2 and 3. Only the independent amplitude defined in (12) and (13) are shown, always as functions of the

⁵Note that the slight differences between the AD1 and AD2 results in the 3rd and 5th column of Table 3, solely come from small differences between the EW scale $\tan \beta$ values. At the grand scale, $\tan \beta = 40$, for both these models.

c.m. energy \sqrt{s} , at a fixed c.m. angle $\theta = 60^\circ$. The upper panels give the HC amplitudes, while the HV amplitudes are shown in the lower panels.

As seen in Figs.2,3, all HV amplitudes vanish at high energies rather quickly. Only for $F_{+-\pm\pm}$ and $F_{-+\pm\pm}$, the power-law vanishing is somewhat delayed by \ln^2 -terms; compare (25) and the discussion just before it.

As the energy increases, the HC amplitudes in Figs.2,3, tend to energy independent, but angle dependent limits; with the approach determined by powers of the c.m. energy, occasionally delayed by logarithmically increasing factors. For $SPS1a'$ and $\theta = 60^\circ$, all amplitudes reach their asymptotic values at around 4 TeV. For other benchmarks, the general shapes will of course remain the same, but the energy where asymptopia is reached may move to higher or lower values, depending on the SUSY scale [8, 9, 10, 23, 24].

In the left upper panel of Fig.4, we present the dimensionless cross sections defined in (4), for $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)$ and $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)$ at $\theta = 60^\circ, 30^\circ, 90^\circ$ in $SPS1a'$. Correspondingly, in the right upper panel, we show $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$ and $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)$ whose asymptotic benchmark dependence only comes from β ; in this case only $\theta = 60^\circ$ is shown.

Unfortunately, for neutralinos, this summed cross section is unobservable, if $\tilde{\chi}_1^0$ is the lightest supersymmetric particle (LSP), which contributes to Dark Matter. For charginos though, it may be observable at sufficient energies. Comparable magnitudes to these summed cross sections should also always appear for $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0)$ and $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$, which are of course observable and benchmark dependent.

The lower panels of Fig.4 intend to show how the various terms in (29, 30, 31) approach 1, as the energy increases. As seen there, we find that for $SPS1a'$ and $\sqrt{s} \gtrsim 5$ TeV,

$$\begin{aligned} \frac{\tilde{\sigma}(gg \rightarrow W^+ W^-)}{\tilde{\sigma}(gg \rightarrow W^+ W^-)^{as}} &\simeq \frac{\tilde{\sigma}(gg \rightarrow ZZ)}{\tilde{\sigma}(gg \rightarrow ZZ)^{as}} \simeq \frac{\tilde{\sigma}(gg \rightarrow \gamma Z)}{\tilde{\sigma}(gg \rightarrow \gamma Z)^{as}} \simeq \frac{\tilde{\sigma}(gg \rightarrow \gamma\gamma)}{\tilde{\sigma}(gg \rightarrow \gamma\gamma)^{as}} \\ &\simeq \frac{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)}{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)^{as}} \simeq \frac{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)}{\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)^{as}} \simeq 1 \quad , \end{aligned} \quad (34)$$

for a wide range of angles. Similar pictures would also hold for all other benchmarks; only the lower bound on energy may occasionally need adjustment, in benchmarks with considerable higher SUSY masses [8, 9, 10, 23, 24]. Apart from the unobservable neutralino member in the last line of (34), this relation is an interesting asymptotic SUSY prediction, which may in fact be valid, even at LHC type energies. Thus, the gauge and chargino members of (34), constitute a prediction, which should in principle be observable. In such cases, the experimental data are to be used for the subprocesses cross sections in the numerators in (34), while the denominators are determined by the simple analytic expressions given in Sect.3.

The situation in the lower panels of Fig.4 changes considerably when specific choices of i, j are made. Thus for $\sqrt{s} \simeq 5$ TeV in $SPS1a'$ and a wide range of angles,

$$\frac{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)}{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)^{as}} \sim 1.4 \quad ; \quad (35)$$

while $\sqrt{s} \gtrsim 20$ TeV are needed for this quantity to approach its asymptotic value of 1. This must be related to the small values of the Z^N matrix elements for $i = 1, j = 2$ in $SPS1a'$, diminishing the contribution of this channel, to the complete dimensionless neutralino production cross section, at a fixed angle; compare upper panels of Fig.4.

The situation is somewhat better for charginos $\tilde{\chi}_1^+ \tilde{\chi}_2^-$ in $SPS1a'$; see lower panel of Fig.4, which roughly satisfies

$$\frac{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)}{\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)^{as}} \sim 1.1 \quad , \quad (36)$$

for the same energies and angles.

5 Concluding remarks

In this paper we have analyzed the helicity amplitudes for $gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j$ at the 1loop EW level in MSSM, and we have observed the validity of the Helicity Conservation (HCns) theorem at sufficient energies. For both, the asymptotically dominant HC amplitudes, and the suppressed HV ones, the approach to their limiting asymptotic values is determined by power law expressions, like m/\sqrt{s} or m^2/s , multiplied by logarithms⁶. During the calculations, it was fascinating to see how the contributions of the various diagrams in Fig.1 were conspiring in order to assure this.

Very simple expressions for the asymptotic $gg \rightarrow \tilde{\chi}_i \tilde{\chi}_j$ HC amplitudes have been written in (26, 27), which depend not only on the gauge couplings and the angle β , but also on ratios of mass-dimension terms entering the chargino and neutralino mixing matrices. The m_t/m_W and m_b/m_W ratios determining the higgsino contributions, also appear in (27).

Combining these results, with the corresponding HC amplitudes for $gg \rightarrow VV'$ derived in [12] and also appearing in (19, 20), the asymptotic subprocess cross section relations in (34) were derived. Particularly interesting among these, are the relations connecting the gauge and total chargino production through gluon-fusion, in (34). Such relations may be testable at LHC type energies, provided the SUSY scale is in the TeV range, as in [6, 7, 8, 9, 10]. Such relations show that HCns may have testable implications at a high energy hadronic collider, even if helicity is not measured directly. Analogous relations for $ug \rightarrow dW^+$ and $ug \rightarrow \tilde{d}_L \tilde{\chi}_i^+$ were derived in [5].

The ggXXcode released here may be used to obtain the corresponding results for any set of real MSSM parameters at the EW scale [14].

Finally, we have also quoted the relations between the asymptotic amplitudes for $gg \rightarrow VV'$ and $\tilde{g}\tilde{g} \rightarrow \tilde{V}\tilde{V}', \tilde{H}_1\tilde{H}_1, \tilde{H}_2\tilde{H}_2$, appearing in (22, 24), which beautifully illustrate how supersymmetry manages to preserve the structure of the amplitudes, in spite

⁶In deriving these results we focused at the sub-sub-leading (i.e. constant) asymptotic contributions the PV expansions [21]. This goes beyond the studies in [25, 26, 27] which only concerned the leading PV parts.

of the fact that the spins of all participating particles are changed.

Summarizing, we reiterate that the work in [1, 2, 4, 5, 11, 12] establishes the helicity conservation theorem (HCns) for any 2-to-2 processes, in the supersymmetric limit of MSSM with R-parity conservation. This limit may be reached by selecting the energy to be much larger than all relevant SUSY masses, while keeping the scattering angle fixed.

If we try to extend these considerations to multibody processes, we immediately realize that complications increase rapidly with the number of external particles. Thus, it is difficult to say something general for the analogous limit, where all subenergy squared and momentum transfers become much larger than the relevant SUSY-masses. Nevertheless, we may claim that, if R-parity is conserved, then all processes involving an odd number of particles must vanish in this supersymmetric limit [1, 2]. Because they will either involve an odd number of sparticles, or their supersymmetric transformed processes will do so.

Thus, only processes involving an even number of particles remain; the least complicated of which, are the 2-to-4 processes. But even for these, the energy needed for making all subenergies and momentum transfers large, becomes prohibitive. Apparently, it is mainly for 2-to-2 processes, that the supersymmetric limit can be studied, at conceivable energies.

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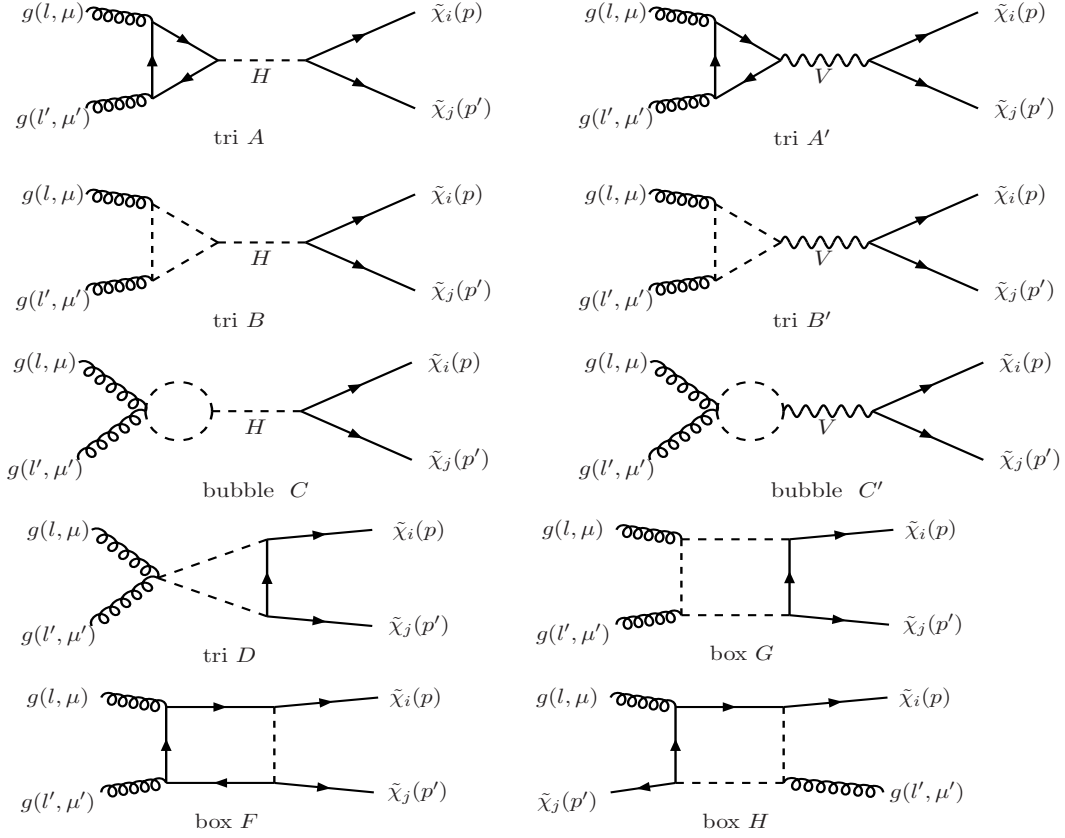


Figure 1: Independent graphs for $gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$, $\tilde{\chi}_i^0 \tilde{\chi}_j^0$. Full external lines describe charginos or neutralinos, while full internal lines denote quark or antiquark exchanges. Broken lines describe squark or antisquark exchanges, or the exchange of a neutral MSSM Higgs-particle denoted as H . Internal wavy lines describe neutral electroweak gauge bosons V .

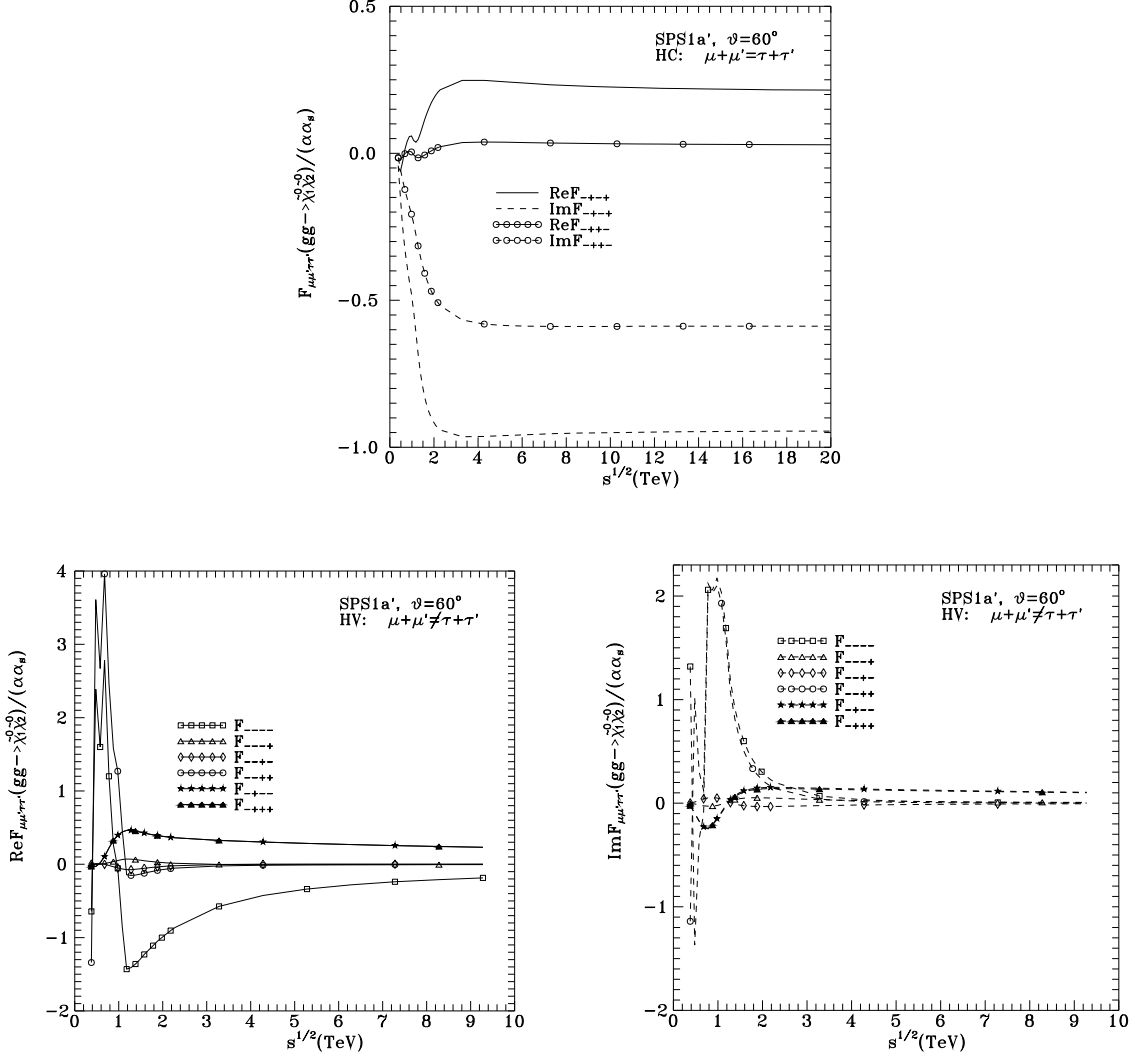


Figure 2: Independent helicity amplitudes for $gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ defined in (12), at $\theta = 60^\circ$ in $SPS1a'$ [6]. The upper panel gives the real and imaginary parts of HC amplitudes, while the lower panels indicate the real and imaginary parts of the HV amplitudes.

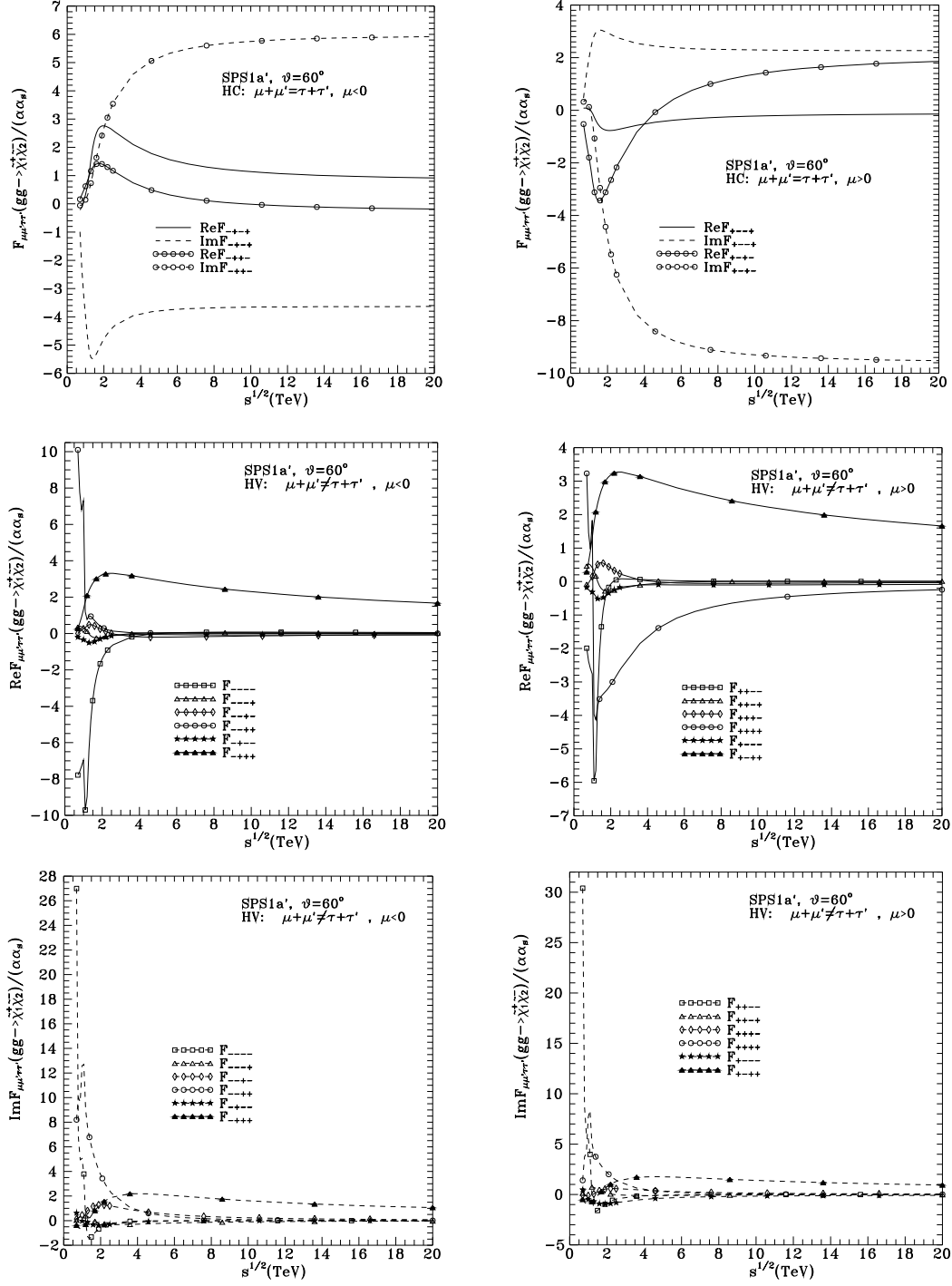


Figure 3: Independent amplitudes for $gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-$ defined in (13), for $SPS1a'$ at $\theta = 60^\circ$. The first row gives the HC amplitudes, while the lower ones describe the real and imaginary parts of the HV amplitudes. Left and right panels describe respectively amplitudes with negative or positive helicity μ for the first gluon.

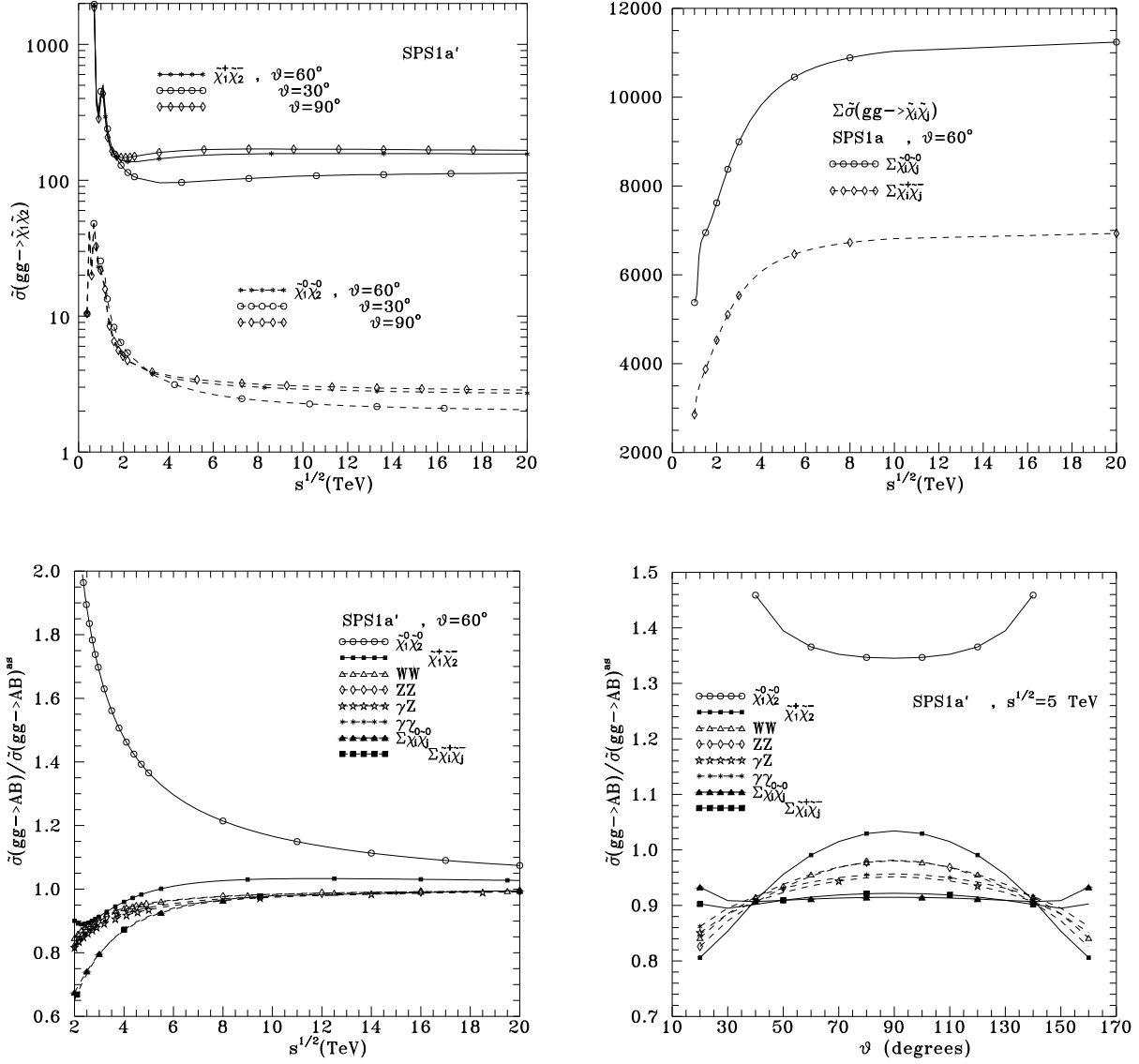


Figure 4: Upper panels: Dimensionless cross sections defined in (4), for $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^-)$ and $\tilde{\sigma}(gg \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0)$ at $\theta = 60^\circ, 30^\circ, 90^\circ$ (left part); and for $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$, $\sum_{ij} \tilde{\sigma}(gg \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0)$ at $\theta = 60^\circ$ (right part). Lower panels: Ratios of these dimensionless cross sections to their asymptotic values, and the corresponding results for $gg \rightarrow VV'$, see (29, 30, 31). Left part displays the energy dependencies of these ratios at $\theta = 60^\circ$, while the right part shows the angular dependencies at $\sqrt{s} = 5\text{TeV}$; compare (34). All in $SPS1a'$.